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Nonstationary Navier-Stokes Problem for Incompressible Fluid with Viscosity

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Abstract Existence and conditional-smooth solution of the Navier-Stokes equation is one of the most important problems in mathematics of the century, which describes the motion of viscous Newtonian fluid and which is a basic of hydrodynamic[1]. Therefore in this work we solve a nonstationary problem Navier-Stokes for incompressible fluid.

Keywords Navier-Stokes, Problem, Conditional-smooth, Solution, Fluid, Flow, Viscosity, Convective the Acceleration, Differentiation, Algorithm, Newton's Potential

1. Introduction

If to designate components of vectors of speed and external force, as

$$v(x,t)=[\upsilon_1(x,t),\upsilon_2(x,t),\upsilon_3(x,t)],$$

$$f(x,t)=[f_1(x,t),f_2(x,t),f_3(x,t)],$$

that for each value $i=1,2,3$ turns out the corresponding scalar equation of Navier-Stokes

$$\frac{\partial \upsilon_i}{\partial t}+\sum_{j=1}^{3}\upsilon_j\frac{\partial \upsilon_i}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\mu\Delta\upsilon_i, \quad (1.1)$$

with conditions

$$\operatorname{div} v=0,((x,t)\in T=R^3\times[0,T_0]) \quad (1.2)$$

$$\upsilon_i\big|_{t=0}=\upsilon_{i0}(x_1,x_2,x_3),\ \forall(x_1,x_2,x_3)\in R^3, \quad (1.3)$$

$\mu>0$ - kinematic viscosity, ρ - density, Δ - Laplas's operator. The additional equation is the condition incompressibility fluid (2). Unknown are speed v and pressure P.

The work purpose. The main object of this work - existence and proofs of single and conditional smoothness of the decision of a problem Navier-Stokes for an incompressible fluid with viscosity.

Theoretical and practical value. Our problem does not include a derivation of an equation in a physical meaning, since there is a big amount of works reflecting these questions[2-4, 8-10]. The Received decisions on the basis of the developed analytical methods proves in the general applicability of the equations of Navier-Stokes.

In a case $0<\mu<1$ the current is considered with very small viscosity. When the current is considered with very small viscosity i.e. when Reynolds's number is very great $(\mathbf{Re}\to\infty)$[8,9] there is an border layer in which viscosity influence is concentrated. In many works in this area of the decision of the equations Navier-Stokes received by the numerical analysis, also confirm these conclusions.

And in a case $0<\mu=\mu_0=\text{const}<+\infty$ the current is considered with average size of viscosity. At very slow currents, or in currents of is strong-viscous liquids of force of a friction much more, than forces of inertia. Hence convective the acceleration doing the equations nonlinear, everywhere are supposed identically equally to a zero[9]. Therefore in a case when convective acceleration is not equal to zero problems connected with methods of integration of the equations of Navier-Stokes in their general view are arisen.

The decision of many problems of theoretical and mathematical physics leads to use of various special weight spaces. In works[5-7] for the first time have offered a method which gives solution of problem Navier-Stokes in $G_\lambda^2(T)$:

$$v\in G_\lambda^2(T)=\{(x_1,x_2,x_3,t)\in T:\upsilon_i\in C^{3,0}(T);$$

$$\upsilon_{it}\in L_\lambda^2(0,T_0),(i=\overline{1,3}),\upsilon_{it}(x_1,x_2,x_3,t)-$$

is continuous and limited functions on

$$(x_1,x_2,x_3)\in R^3,\ C^{3,3,3,0}(T)\equiv C^{3,0}(T)\}$$

and

$$\begin{cases}\|v\|_{G_\lambda^2(T)}=\sum\limits_{i=1}^{3}\|\upsilon_i\|_{\tilde D^2_{(\upsilon_i;\lambda)}(T)},\\ \|\upsilon_i\|_{\tilde D^2_{(\upsilon_i;\lambda)}(T)}=\|\upsilon_i\|_{C^{3,0}(T)}+\|\upsilon_{it}\|_{L_\lambda^2},i=\overline{1,3},\end{cases} \quad (1.4)$$

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$$\begin{cases} \|\upsilon_{it}\|_{L^2_\lambda} = (\sup_{R^3} \int_0^{T_0} \lambda(t)|\upsilon_{it}(x_1,x_2,x_3,t)|^2\, dt)^{\frac{1}{2}}, \\ 0 \le \lambda(t): \int_0^{T_0} \lambda(t)\frac{1}{t}dt = q_0. \end{cases}$$

To answer the brought attention to the question, we offer the following method of the decision of a problem Navier-Stokes. For that the phrpose, system (1.1) we will transform to a kind

$$\upsilon_{it} + \theta_i = f_i - \frac{1}{\rho}P_{x_i} - \frac{1}{2}Q_{x_i} + \mu\Delta\upsilon_i, (i=\overline{1,3}), \quad (1.5)$$

$$\theta_i = \sum_{j=1}^{3}(\upsilon_j\upsilon_{ix_j} - \frac{1}{2}Q_{x_i}), (i=\overline{1,3}), \quad (1.6)$$

$$\theta_i\big|_{t=0} = \theta_i^0(x_1,x_2,x_3), \forall(x_1,x_2,x_3)\in R^3, \quad (1.7)$$

$$\begin{cases} Q \equiv \sum_{i=1}^{3}\upsilon_i^2,\ Q_{x_i} = 2\sum_{j=1}^{3} \upsilon_j\upsilon_{jx_i}, (i=\overline{1,3}), \\ Q^0_{x_i} = [\sum_{j=1}^{3}\upsilon^2_{j0}]_{x_i} = 2\sum_{j=1}^{3} \upsilon_{j0}\upsilon_{j0x_i}, \end{cases} \quad (1.8)$$

without breaking equivalence of system (1.1) and (1.5), (1.6). The received systems (1.5), (1.6) contain unknown persons υ_i, $\theta_i, (i=\overline{1,3})$ and pressure P. Here θ_i^0 -known functions because are known $\upsilon_{j0}, \upsilon_{j0x_i}$.

The developed method of the decision of systems (1.5) and (1.6), is connected with functions $\theta_i, (i=\overline{1,3})$, i.e.

A) $\mathrm{rot}\tilde{\theta} = 0, \tilde{\theta} = (\theta_1,\theta_2,\theta_3); \mathrm{rot}\, v \ne 0$ or

B) $\theta_i, (i=\overline{1,3})$ - any functions if, accordingly, as necessary conditions, take place:

a0) $\mathrm{rot}\tilde{\theta}^0 = 0, \tilde{\theta}^0 = (\theta_1^0,\theta_2^0,\theta_3^0)$, b0) $\tilde{\theta}^0$ - any functions.

2. A Problem of Navier-Stokes with a Condition (A)

In this paragraph in the subsequent points, at the specified restrictions on the entrance data, the strict substantiation of compatibility of systems (1.5), (1.6) will be given.

2.1. Research With a Condition (A)

Let functions $\theta_i^0, (i=\overline{1,3})$ satisfy to a condition (a0). Then relatively $\theta_i, (i=\overline{1,3})$ we suppose a condition (A) and

$$\mathrm{div}f \ne 0, 0 < \mu < 1, \quad (2.1)$$

where from system (1.5) and (1.6), accordingly we will receive following systems

$$\upsilon_{it} + \theta_{x_i} + \frac{1}{2}Q_{x_i} = f_i - \frac{1}{\rho}P_{x_i} + \mu\Delta\upsilon_i, (i=\overline{1,3}), \quad (2.2)$$

$$\begin{cases} \theta_i = \theta_{x_i}, \\ \theta_{x_i} = \sum_{j=1}^{3}(\upsilon_j\upsilon_{ix_j} - \frac{1}{2}Q_{x_i}), (i=\overline{1,3}). \end{cases} \quad (2.3)$$

Theorem 1. Let conditions (1.2), (1.3), (A) and (2.1) are satisfied. Then systems (2.2) and (2.3) it is equivalent will be transformed to a kind

$$\begin{cases} \Delta J = -F_0,\ J \equiv \frac{1}{\rho}P + \frac{1}{2}Q + \theta,\ F_0 \equiv -\sum_{i=1}^{3} f_{ix_i}, \\ \upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_{x_i}, \\ \Delta\theta = -\psi^0,\ \psi^0 \equiv -\sum_{i=1}^{3} \psi_{ix_i}(x_1,x_2,x_3,t), \\ \frac{1}{\rho}P = -\frac{1}{2}Q - \theta + \\ +\frac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3,t)\frac{ds_1ds_2ds_3}{r}, \\ r = \sqrt{(x_1-s_1)^2 + (x_2-s_2)^2 + (x_3-s_3)^2}. \end{cases} \quad (2.4)$$

Hence, the problem (1.1) - (1.3) has the unique decision which satisfies to a condition (1.2).

Proof. From system (2.2) it is visible, if the 1-equation (2.2, $i=1$) it is differentiated on x_1, 2-equation on x_2 (2.2, $i=2$), 3-equation on x_3 (2.2, $i=3$), and it is summarised, we will receive the equation of Puasson[10]

$$\Delta J = -F_0, \quad (2.5)$$

as

$$\begin{cases} \frac{\partial}{\partial t}(\upsilon_{1x_1} + \upsilon_{2x_2} + \upsilon_{3x_3}) + \Delta(\frac{1}{2}Q + \theta + \frac{1}{\rho}P) = \\ = \sum_{i=1}^{3} f_{ix_i} + \mu\sum_{i=1}^{3}\frac{\partial^2}{\partial x_i^2}(\upsilon_{1x_1} + \upsilon_{2x_2} + \upsilon_{3x_3}), \\ \mathrm{div}\, v = 0, \mathrm{div} f = -F_0. \end{cases}$$

At that it is proved

$$J = \frac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3;t)\frac{ds_1ds_2ds_3}{r}, \quad (2.6)$$

$$J_{x_i} = \frac{1}{4\pi}\int_{R^3} \tau_i F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t)\times$$
$$\times\frac{d\tau_1 d\tau_2 d\tau_3}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}, (s_i - x_i = \tau_i, i=\overline{1,3}). \quad (2.7)$$

Algorithm when we will receive the equation of Puasson (2.5) for brevity we name «algorithm puassonization systems».

In work of Sobolev[10] it is specified that function (2.6) satisfies to the equation (2.5) and is called Newtons' potential.

Therefore, if J - the decision of the equation (2.5), then substituting

$$J_{x_i} \underset{\overline{x}}{\equiv} \frac{1}{\rho} P_{x_i} + \frac{1}{2} Q_{x_i} + \theta_{\,i} \tag{2.8}$$

in (2.2), we have

$$\upsilon_{it} = f_i + \mu \Delta \upsilon_i - J_i, (i = \overline{1,3}; J_{x_i} \equiv J_i), \tag{2.9}$$

i.e. system (2.2) it is equivalent by (2.9) will be transformed to a kind linear the nonuniform equation of heat conductivity. The equations (2.5), (2.9) is there are first and second equations of system (2.4).

The system (2.9) is solved by S.L.Sobolev's method:

$$\upsilon_i = \frac{1}{8(\sqrt{\pi \mu t})^3} \int_{R^3} \exp(-\frac{r^2}{4\mu t}) \upsilon_{i0}(s_1, s_2, s_3) \times$$

$$\times ds_1 ds_2 ds_3 + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-s)}) \times$$

$$\times \frac{1}{\sqrt{(\mu(t-s))^3}} [f_i(s_1, s_2, s_3, s) - J_i(s_1, s_2, s_3, s)] \times$$

$$\times ds_1 ds_2 ds_3 ds \equiv \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \times$$

$$\times \upsilon_{i0}(x_1 + 2\tau_1\sqrt{\mu t}, x_2 + 2\tau_2\sqrt{\mu t}, x_3 + 2\tau_3\sqrt{\mu t}) \times$$

$$\times d\tau_1 d\tau_2 d\tau_3 + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \times$$

$$\times \big[f_i(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)},$$

$$x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) - J_i(x_1 + 2\tau_1\sqrt{\mu(t-s)},$$

$$x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) \big] \times$$

$$\times d\tau_1 d\tau_2 d\tau_3 ds \equiv H_i, i = \overline{1,3}, \tag{2.10}$$

where

$$s_i - x_i = \tau_i 2\sqrt{\mu t} \text{ or } s_i - x_i = \tau_i 2\sqrt{\mu(t-s)}.$$

All H_i - is known functions and

$$\upsilon_{ix_j}, (i = \overline{1,3}, j = \overline{1,3})$$

are defined from system (2.10):

$$\upsilon_{ix_j} = \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \upsilon_{i0x_j}(x_1 +$$

$$+2\tau_1\sqrt{\mu t}, x_2 + 2\tau_2\sqrt{\mu t}, x_3 + 2\tau_3\sqrt{\mu t}) d\tau_1 d\tau_2 d\tau_3 +$$

$$+\frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \Big[f_{ix_j}(x_1 + 2\tau_1 \times$$

$$\times \sqrt{\mu(t-s)}, x_2 + 2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3 \times$$

$$\times \sqrt{\mu(t-s)}; s) - J_{ix_j}(x_1 + 2\tau_1\sqrt{\mu(t-s)}, x_2 +$$

$$+2\tau_2\sqrt{\mu(t-s)}, x_3 + 2\tau_3\sqrt{\mu(t-s)}; s) \Big] \times$$

$$\times d\tau_1 d\tau_2 d\tau_3 ds \equiv H_{ix_j}, i = \overline{1,3}, j = \overline{1,3}. \tag{2.11}$$

Then, on the basis of (2.3), (2.10) and (2.11), and their private derivatives on x_i, we find

$$\theta_{x_i} = \sum_{j=1}^{3} (H_j \cdot H_{ix_j} - H_j \cdot H_{jx_i}) \equiv \psi_i, i = \overline{1,3}. \tag{2.12}$$

As ψ_i - is known functions, hence from system (2.12) differentiating 1 equation on x_1[(2.12): $i=1$], 2 equations on x_2[(2.12): $i=2$], 3 equations on x_3[(2.12): $i=3$], and summarising, we will receive

$$\Delta\theta = -\psi^0, \psi^0 \equiv -\sum_{i=1}^{3} \psi_{ix_i}, \tag{2.13}$$

at that

$$\theta \in C^2(T) : \theta = \frac{1}{4\pi} \int_{R^3} \psi^0(s_1, s_2, s_3, t) \frac{ds_1 ds_2 ds_3}{r}.$$

The equation (2.13) is the third equation of system (2.4). Therefore, from the received results, taking into account (2.6), follows

$$\frac{1}{\rho} P = -\theta - \frac{1}{2} Q + a + \frac{1}{4\pi} \int_{R^3} F_0(s_1, s_2, s_3, t) \frac{ds_1 ds_2 ds_3}{r}, \tag{2.14}$$

i.e. functions υ_i, θ, P are defined from systems (2.10), (2.13), (2.14).

Uniqueness is obvious, as a method by contradiction from (2.10) uniqueness of the decision follows $\upsilon_i \in ?^{3,0}\ T)$, $i = \overline{1,3}$. Results (2.10) with a condition ((A), (2.1)) are received where smoothness of functions υ_i is required only on x_i as the derivative of 1st order in time has feature in $t=0$. Then taking into account (2.10), (2.13), (2.14) and the system (2.4) has the unique continuous decision.

Further, considering private derivatives of 1st order

$$\upsilon_{x_i} = \frac{\partial}{\partial x_i} \{H_i\}, i = \overline{1,3}, \tag{2.15}$$

and summarising (2.15) with taking into account (1.2), we have

$$0=\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp[-(\tau_1^2+\tau_2^2+\tau_3^2)]\{-F_0[x_1+$$
$$+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+$$
$$+2\tau_3\sqrt{\mu(t-s)};s]-\Delta J[x_1+2\tau_1\sqrt{\mu(t-s)},$$
$$x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s]\}\times$$
$$\times d\tau_1 d\tau_2 d\tau_3 ds=0,$$

as $\Delta J=-F_0$.

Means, the system (2.10) satisfies to the equation (1.2).

2.2. Limitation of Functions $(\upsilon_1,\upsilon_2,\upsilon_3)$ in $G_\lambda^2(T)$

The limiting case which we will consider concerns results of the theorem 1. Then the decision of system (1.1) is representing in the form of (2.10) with conditions (1.2), (1.3), (A), (2.1) and

$$\begin{cases} f_i:\sup\limits_{R^3}\int_0^{T_0}\left|D^k f_i(x_1,x_2,x_3,s)\right|ds\le\beta_1, \\ \sup\limits_T\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\frac{1}{\sqrt{t-s}}\sum_{j=1}^3|\tau_j|\times\left|f_{il_j}(l_1,l_2,l_3;s)\right|ds\le\beta_2, \\ (\sup\limits_{R^3}\int_0^{T_0}\lambda(s)\left|f_i(x_1,x_2,x_3,s)\right|^2ds)^{\frac{1}{2}}\le\beta_3, \\ J_{x_i}\equiv J_i:\sup\limits_{R^3}\int_0^{T_0}\left|D^k J_i(x_1,x_2,x_3,s)\right|ds\le\beta_4, \\ (\sup\limits_{R^3}\int_0^{T_0}\lambda(s)\left|J_i(x_1,x_2,x_3,s)\right|^2ds)^{\frac{1}{2}}\le\beta_5, \\ \sup\limits_T\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\frac{1}{\sqrt{t-s}}\sum_{j=1}^3|\tau_j|\times\left|J_{il_j}(l_1,l_2,l_3;s)\right|ds\le\beta_6, \\ \upsilon_{i0}:\sup\limits_{R^3}\left|D^k\upsilon_{i0}\right|\le\beta_7,(i=1,2,3;\ j=1,2,3; \\ k=0,1,2,3),l_j=x_j+2\tau_j\sqrt{\mu(t-s)}, \\ \beta=\max\limits_{1\le i\le 7}\beta_i;\ \beta_0=\beta(3\sqrt{\mu q_0}+2+2\sqrt{\mu q_1}), \\ \int_0^{T_0}\lambda(t)dt=q_1,\ \int_0^{T_0}\lambda(t)\frac{1}{t}dt=q_0. \end{cases} \tag{2.16}$$

Really, estimating (2.10) in $G_\lambda^2(T)$, we have

$$\begin{cases} \|\nu\|_{G_\lambda^2(T)}\le 3[N_1+\beta_0]=M^*, \\ \|\upsilon_i\|_{C^{3,0}(T)}=\sum\limits_{0\le|k|\le 3}\left\|D^k\upsilon_i\right\|_{C(T)}\le N_1=60\beta, \end{cases}$$

$$\begin{cases} \|\upsilon_i\|_{C(T)}\le 3\beta,\ (\beta_1+\beta_4+\beta_7\le 3\beta),i=\overline{1,3}, \\ \|\upsilon_{it}\|_{L_\lambda^2}\le\beta(3\sqrt{\mu q_0}+2+2\sqrt{\mu q_1})=\beta_0, \\ k=0:D^0\upsilon_i\equiv\upsilon_i;k\ne 0:D^k\upsilon_i= \\ =\frac{\partial^{|k|}\upsilon_i}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\partial x_3^{\alpha_3}},|k|=\sum\limits_{i=1}^3\alpha_i,(\alpha_i=\overline{0,3}). \end{cases} \tag{2.17}$$

Theorem 2. In the conditions of the theorem 1 and (2.16), (2.17) the problem (1.1) - (1.3) has the unique decision in $G_\lambda^2(T)$.

3. The Decision of a Problem of Navier-Stokes with a Condition (B)

Here we investigate a case (B) when $\theta_i,(i=\overline{1,3})$ containing convective members of a problem of Navier-Stokes are any. Results of the theorem 1 are not applicable.

Therefore, for the decision of a problem (1.1) - (1.3) we offer following algorithms.

3.1. Problem Navier- Stokes with Average Viscosity

Let conditions (1.2), (1.3) are satisfied and:

$$\begin{cases} \upsilon_i(x_1,x_2,x_3,0)=\upsilon_{i0}(x_1,x_2,x_3)\equiv V_i^0(\xi), \\ \xi=\sum\limits_{i=1}^3\gamma_i x_i;R\ni\gamma_i:\ \sum\limits_{i=1}^3\gamma_i=0,k=\sum\limits_{i=1}^3\gamma_i^2, \end{cases} \tag{3.1}$$

at that

$$\begin{cases} \upsilon_i\equiv V_i(\xi,t),(i=\overline{1,3}), \\ V_i(\xi,0)=V_i^0(\xi);\mathrm{div}f\ne 0,0<\mu=\mu_0, \\ \sum\limits_{i=1}^3\gamma_i V_{\xi^m}(\xi,t)=0,(m=\overline{1,3}). \end{cases} \tag{3.2}$$

Then on the basis of functions $V_i(\xi,t),(i=\overline{1,3})$ and

$$\begin{cases} P_{x_i}(x_1,x_2,x_3,t)=\gamma_i P_\xi(\xi,t), \\ \upsilon_{it}(x_1,x_2,x_3,t)=V_{it}(\xi,t),(i=\overline{1,3}), \\ \upsilon_{ix_j}(x_1,x_2,x_3,t)=\gamma_j V_{i\xi}(\xi,t), \\ \upsilon_{ix_j^2}(x_1,x_2,x_3,t)=\gamma_j^2 V_{i\xi^2}(\xi,t);\mu\Delta\upsilon_i=\mu k V_{i\xi^2}, \end{cases}$$

system (1.1) it is equivalent will be transformed to a kind

$$\begin{cases} LV_i \equiv V_{it}(\xi,t)+Z(\xi,t)\times V_{i\xi}(\xi,t)= \\ = f_i(\xi,t)-\dfrac{1}{\rho}\gamma_i P_\xi(\xi,t)+k\mu V_{i\xi^2},(i=\overline{1,3}), \\ Z(\xi,t)\equiv\sum_{i=1}^{3}\gamma_i V_i(\xi,t); Z_\xi=0. \end{cases} \quad (3.3)$$

In the specified systems unknown persons contain V_i, P.

Remark 1. Under regular in

$D_0=\{(\xi,t):\xi\in R, 0<t\le T_0\}$

the decision we understand the decision $V_i, i=1,2,3$ the equation (1.1) in D_0, which has a continuous derivative on ξ to the third order inclusive and continuous derivative on *t(t>0)*.

From system (3.3), considering conditions (3.2), and having entered «algorithm puassonization systems», i.e. differentiating the equations of system (3.3) $\times\gamma_i$ accordingly on ξ and, then summarising, we have the equation:

$$\begin{cases} \dfrac{1}{\rho}P_{\xi^2}=F_0(\xi,t)\equiv\dfrac{1}{k}\sum_{i=1}^{3}\gamma_i f_{i\xi}(\xi,t), \\ P_\xi^{(n)}(\xi,t)\big|_{\xi\to\infty}=0,(n=0,1), \\ \xi F_1(\xi,t)\big|_{\xi\to\infty}=0,(F_1(\xi,t)\equiv\dfrac{1}{k}\sum_{i=1}^{3}\gamma_i f_i(\xi,t)). \end{cases} \quad (3.4)$$

Therefore, we will receive

$$\begin{cases} \dfrac{1}{\rho}P=\int_\xi^{+\infty}(\eta-\xi)\dfrac{1}{k}\sum_{i=1}^{3}\gamma_i f_{i\eta}(\eta,t)d\eta= \\ =-\int_\xi^{+\infty}\dfrac{1}{k}\sum_{i=1}^{3}\gamma_i f_i(\eta,t)d\eta=-\int_\xi^{+\infty}F_1(\eta,t)d\eta, \\ \dfrac{1}{\rho}P_\xi=F_1(\xi,t);\sum_{i=1}^{3}\gamma_i F_1=0. \end{cases} \quad (3.5)$$

Really on a basis[(3.3): $\dfrac{\partial}{\partial\xi}\gamma_i\times(3.3)$], we have

$$\begin{cases} \sum_{i=1}^{3}\gamma_i\dfrac{\partial}{\partial\xi}(LV_i)(\xi,t)=\sum_{i=1}^{3}\gamma_i f_{i\xi}(\xi,t)- \\ -\dfrac{1}{\rho}kP_{\xi^2}(\xi,t)+k\mu\sum_{\xi=1}^{3}\gamma_i V_{i\xi^3}, \\ Z_\xi(\xi,t)=0. \end{cases}$$

Then we have the following (3.4).

Further, we have

$$\begin{cases} V_{it}+Z\times V_{i\xi}=\Phi_i+k\mu V_{i\xi^2}, i=\overline{1,3}, \\ \Phi_i(\xi,t)\equiv f_i(\xi,t)-\gamma_i F_1(\xi,t), \end{cases} \quad (3.6)$$

or for consideration of unknown functions V_i we have

$$\begin{aligned} V_i=&\frac{1}{\sqrt{\pi}}\int_R\exp(-\tau^2)V_i^0(\xi+2\tau\sqrt{\alpha t})dt+\frac{1}{\sqrt{\pi}}\times \\ &\times\int_0^t\int_R\exp(-\tau^2)\Phi_i(\xi+2\tau\sqrt{\alpha(t-s)};s)d\tau ds+ \\ &+\frac{1}{\sqrt{\pi}}\int_0^t\int_R\exp(-\tau^2)(\sum_{j=1}^{3}\gamma_j V_j(\xi+2\tau\times \\ &\times\sqrt{\alpha s};t-s))\frac{\tau}{\sqrt{\alpha s}}V_i(\xi+2\tau\sqrt{\alpha s};t-s)\times \\ &\times d\tau ds\equiv D_i[V_1,V_2,V_3],(i=\overline{1,3};\alpha=k\mu), \end{aligned} \quad (3.7)$$

as here consider a method integration in parts

$$\begin{cases} 1)-\dfrac{1}{2\sqrt{\pi}}\int_0^t\int_R(\exp(-\dfrac{(\xi-\eta)^2}{4\mu k(t-s')}))\dfrac{1}{\sqrt{\mu k(t-s')}}\times \\ \times(\sum_{j=1}^{3}\gamma_j V_j(\eta,s'))\times V_{i\eta}(\eta,s')d\eta ds'=\dfrac{1}{\sqrt{\pi}}\times \\ \times\int_0^t\int_R\exp(-\tau^2)(\sum_{j=1}^{3}\gamma_j V_j(\xi+2\tau\sqrt{\mu ks};t-s))\times \\ \times\dfrac{\tau}{\sqrt{\mu ks}}V_i(\xi+2\tau\sqrt{\mu ks};t-s)d\tau ds, \\ (\eta-\xi=2\tau\sqrt{\mu k(t-s')};t-s'=s), \\ Z_\eta(\eta,s')=\sum_{j=1}^{3}\gamma_j V_{j\eta}(\eta,s')=0, \end{cases}$$

and

$$\begin{cases} 2)\ \dfrac{1}{2\sqrt{\pi\mu kt}}\int_R\exp(-\dfrac{(\xi-\eta)^2}{4\mu kt})V_i^0(\eta)d\eta+ \\ +\dfrac{1}{2\sqrt{\pi}}\int_0^t\int_R\dfrac{1}{\sqrt{\mu k(t-s)}}\exp(-\dfrac{(\xi-\eta)^2}{4\mu k(t-s)})\times \\ \times\Phi_i(\eta,s)d\eta ds=\dfrac{1}{\sqrt{\pi}}\int_R\exp(-\tau^2)V_i^0(\xi+ \\ +2\tau\sqrt{\mu kt})dt+\dfrac{1}{\sqrt{\pi}}\int_0^t\int_R\exp(-\tau^2)\Phi_i(\xi+ \\ +2\tau\sqrt{\mu k(t-s)};s)d\tau ds, \\ (\eta-\xi=2\tau\sqrt{\mu kt}\text{ or }\eta-\xi=2\tau\sqrt{\mu k(t-s)}). \end{cases}$$

If functions $V_i, i=\overline{1,3}$ are system decisions (3.7) thus takes place (3.2) and

$$\begin{cases} \forall(\xi,t)\in\bar{D}_0=\{(\xi,t):\xi\in R,0\le t\le T_0\}, \\ \Phi_i,V_i^0:\frac{1}{\sqrt{\pi}}\sup_{\bar{D}_0}\int_0^t\int_R\exp(-\tau^2)\left|\Phi_{il^k}^{(k)}(\xi+2\tau\times\right. \end{cases}$$

$$\begin{cases} \left.\times\sqrt{\alpha(t-s)};s)\right|d\tau ds\le\beta_1, \\ \Phi_{il^0}^{(0)}\equiv\Phi_i;(\xi,t)\in\bar{D}_0:|V_i(\xi,t)|\le r_0, \\ \sup_{\bar{D}_0}\frac{1}{\sqrt{\pi}}\int_0^t\int_R\exp(-\tau^2)|\tau|\frac{1}{\sqrt{s}}\sum_{j=1}^3|\gamma_j|\times \\ \times r_0 d\tau ds\le\beta_2 r_0, \\ \sup_R\left|V_i^{0(k)}\right|\le\beta_3,(i=\overline{1,3};k_0=\overline{0,3}), \\ l=\xi+2\tau\sqrt{\alpha(t-s)};\beta=\max_{1\le i\le3}\beta_i, \\ \alpha=k\mu;0<\mu=\mu_0, \\ d=\sum_{i=1}^3 d_i<1,(d_i=2\beta r_0\frac{1}{\sqrt{\alpha}},i=\overline{1,3}), \end{cases} \quad (3.8)$$

that $V_i\in C^{3,0}(\bar{D}_0)$:

$$\begin{cases} E=\sum_{i=1}^3\|V_i(\xi,t)\|_C: \\ E\le(1-d)^{-1}6\beta=M_0. \end{cases} \quad (3.9)$$

Then the solution of this system (3.7) we can find on the basis of Pikard's method

$$V_{i,n+1}=D_i[V_{1,n},V_{2,n},V_{3,n}],n=0,1,...,(i=\overline{1,3}), \quad (3.10)$$

where $V_{i,0},i=\overline{1,3}$ - initial estimates and at that

$$\begin{cases} \forall(\xi,t)\in\bar{D}_0,V_i:|V_i-V_{i,0}|\le r=\text{const}, \\ E_0=\sum_{i=1}^3\|V_i-V_{i,0}\|,E_{n+1}=\sum_{i=1}^3\|V_i-V_{i,n+1}\|, \\ E_{n+1}\le d^{n+1}E_0\xrightarrow[n\to\infty]{d<1}0, \\ V_{i,n}\xrightarrow[n\to\infty]{d<1}V_i\equiv H_i^0\in C^{3,0}(\bar{D}_0),(i=\overline{1,3}). \end{cases} \quad (3.11)$$

Theorem 3. Under conditions (2), (3), (3.2), (3.8) problem Navier-Stokes has the unique continuous decision.

Definition 1. The generalised decision a problems (1.1)-(1.3), (3.2) in area D_0 we name any continuous in $\bar{D}_0$ equation decision (3.7), when $0<\mu=\mu_0$.

3.2. We will Consider a Fluid with Very Small Viscosity

Let conditions (1.2), (1.3), (3.1) are satisfied and:

$$\begin{cases} \upsilon_i\equiv V_i(\xi,t)+K_0(\xi,t),(i=\overline{1,3}), \\ K_0(\xi,t)\equiv\sqrt{(\mu kt)^3}\exp(-\frac{\xi^2}{4k\mu t}), \\ V_i(\xi,0)=V_i^0(\xi), \end{cases}$$

$$\begin{cases} \operatorname{div}f\ne0;0<\mu<1;P_\xi(\xi,t)|_{\xi\to\infty}=0, \\ \sum_{i=1}^3\gamma_i V_{i\xi^m}^{(m)}(\xi,t)=0,(m=\overline{0,2}),\forall(\xi,t)\in\bar{D}_0, \\ V_{i\xi^0}^{(0)}\equiv V_i,(m=0:Z\equiv\sum_{i=1}^3\gamma_i V_i=0), \\ \tilde{V}=(V_1,V_2,V_3),\tilde{V}^0=(V_1^0,V_2^0,V_3^0), \end{cases} \quad (3.12)$$

at that

$$\begin{cases} P_{x_i}(x_1,x_2,x_3,t)=\gamma_i P_\xi(\xi,t), \\ \upsilon_{it}(x_1,x_2,x_3,t)=V_{it}(\xi,t)+\sqrt{(\mu kt)^3}\frac{\xi^2}{4k\mu t^2}\times \\ \times\exp(-\frac{\xi^2}{4k\mu t})+\frac{3}{2}\sqrt{(\mu k)^3 t}\times\exp(-\frac{\xi^2}{4k\mu t}), \\ \upsilon_{ix_j}(x_1,x_2,x_3,t)=\gamma_j V_{i\xi}(\xi,t)-\sqrt{(\mu kt)^3}\gamma_j\times \\ \times\frac{2\xi}{4k\mu t}\exp(-\frac{\xi^2}{4k\mu t}), \\ \upsilon_{ix_j^2}(x_1,x_2,x_3,t)=\gamma_j^2 V_{i\xi^2}(\xi,t)+\sqrt{(\mu kt)^3}\gamma_j^2\times \\ \times\frac{\xi^2}{4k^2\mu^2t^2}\exp(-\frac{\xi^2}{4k\mu t})-\gamma_j^2\sqrt{(\mu k)^3 t}\times \\ \times\frac{1}{2k\mu}\exp(-\frac{\xi^2}{4k\mu t}), \\ \mu\Delta\upsilon_i=\mu kV_{i\xi^2}+\sqrt{(\mu kt)^3}\frac{\xi^2}{4k\mu t^2}\exp(-\frac{\xi^2}{4k\mu t})- \\ -\frac{1}{2}\sqrt{(\mu k)^3 t}\times\exp(-\frac{\xi^2}{4k\mu t}), \\ \sum_{j=1}^3\upsilon_j\frac{\partial\upsilon_i}{\partial x_j}=[\sum_{j=1}^3\gamma_j(V_i(\xi,t)+\sqrt{(\mu kt)^3}\times \\ \times\exp(-\frac{\xi^2}{4k\mu t}))]\times[V_{i\xi}(\xi,t)-\sqrt{(\mu kt)^3}\frac{2\xi}{4k\mu t}\times \\ \times\exp(-\frac{\xi^2}{4k\mu t})]=0,(i=\overline{1,3};Z=0). \end{cases}$$

Then on the basis of functions $V_i(\xi,t),(i=\overline{1,3})$, system (1.1) it is equivalent will be transformed to a kind:

$$LV_i \equiv V_{it} + 2\sqrt{(\mu k)^3 t}\times\exp(-\frac{\xi^2}{4k\mu t}) =$$

$$= f_i(\xi,t) - \frac{1}{\rho}\gamma_i P_\xi(\xi,t) + k\mu V_{i\xi^2},(i=\overline{1,3}), \quad (3.13)$$

in the specified systems unknown persons contain V_i, P. Therefore

$$\begin{cases} \frac{1}{\rho}P_\xi(\xi,t) = F_1 \equiv \frac{1}{k}\sum_{i=1}^{3}\gamma_i f_i(\xi,t), \\ \frac{1}{\rho}P = -\int_\xi^{+\infty} F_1(\eta,t)d\eta, \forall(\xi,t)\in\overline{D}_0, \end{cases} \quad (3.14)$$

as

$$\frac{\partial}{\partial t}(\sum_{i=1}^{3}\gamma_i V_i) + 2\sum_{i=1}^{3}\gamma_i\sqrt{(\mu k)^3 t}\times\exp(-\frac{\xi^2}{4k\mu t}) =$$

$$= \sum_{i=1}^{3}\gamma_i f_i(\xi,t) - \frac{1}{\rho}kP_\xi(\xi,t) + k\mu\sum_{i=1}^{3}\gamma_i V_{i\xi^2},(i=\overline{1,3}).$$

Theorem 4. Let functions V_i are system decisions

$$\begin{cases} V_{it}(\xi,t) = \Phi_i(\xi,t) + k\mu V_{i\xi^2},(i=\overline{1,3}), \\ \Phi_i \equiv f_i - \gamma_i F_1 - 2\sqrt{t(\mu k)^3}\exp(-\frac{\xi^2}{4k\mu t}). \end{cases} \quad (3.15)$$

Then

$$V_i(\xi,t) = \frac{1}{\sqrt{\pi}}\int_R \exp(-\tau^2)V_i^0(\xi+2\tau\sqrt{\alpha t})d\tau +$$

$$+\frac{1}{\sqrt{\pi}}\int_0^t\int_R \exp(-\tau^2)\Phi_i(\xi+2\tau\sqrt{\alpha(t-s)};s)d\tau ds \equiv$$

$$\equiv H_i(\xi,t), i=\overline{1,3}; \alpha = k\mu, \quad (3.16)$$

where known functions and $H_i \in G_\lambda^2(D_0)$:

$$\begin{cases} \|\tilde{V}\|_{G_\lambda^2(D)} = \sum_{i=1}^{3}(\|V_i\|_{C^{3,0}} + \|V_{it}\|_{L_\lambda^2}) \le M, \\ \|V_i\|_{C^{3,0}} = \sum_{k=0}^{3}\left\|H_{i\xi^m}^{(m)}\right\|_C \le M_1, \\ \|V_{it}\|_{L_\lambda^2} = (\sup_R \int_0^{T_0}\lambda(s)|H_{it}(\xi,s)|^2 ds)^{\frac{1}{2}} \le M_2, \\ (\sup_{\overline{D}_0}\int_0^t \lambda(s)|\Phi_i(\xi,s)|^2 ds)^{\frac{1}{2}} \le M_3, \\ H_{i\xi^0}^{(0)} \equiv H_i,(i=\overline{1,3}; k=\overline{0,3}), \\ q_0 = \int_0^{T_0}\lambda(t)\frac{1}{t}dt, q_1 = \int_0^{T_0}\lambda(t)dt. \end{cases}$$

Theorem 5. If functions $V_i, P, i=\overline{1,3}$ are system decisions (3.14), (3.15), (3.10) that (3.1) is the decision of system (1.1) in $G_\lambda^2(D_0)$:

$$\upsilon_i = H_i(\xi,t) + \sqrt{(\mu kt)^3}\exp(-\frac{\xi^2}{4k\mu t}). \quad (3.17)$$

Remark 2. In particular, if conditions (1.2), (1.3), (3.12) and

$$\begin{cases} K_0(\xi,t) \equiv 0: \\ \upsilon_i = V_i(\xi,t), i=\overline{1,3}, \end{cases} \quad (3.18)$$

are satisfied. Then takes place

$$V_{it} = f_i(\xi,t) - \frac{1}{\rho}\gamma_i P_\xi + \mu k V_{i\xi^2}, i=\overline{1,3}. \quad (3.19)$$

From here, considering (3.14), (3.15) we have

$$\begin{cases} \frac{1}{\rho}P = -\int_\xi^\infty F_1(\eta,t)d\eta, \\ V_{it} = \Phi_i + \mu k V_{i\xi^2}, \\ \Phi_i(\xi,t) \equiv f_i(\xi,t) - \gamma_i F_1(\xi,t), i=\overline{1,3}, \\ \sum_{i=1}^{3}\gamma_i F_1 = 0. \end{cases} \quad (3.20)$$

Hence all conditions of the theorem 4 and 5 are satisfied.

Example 1. The specified method of the theorem 5 can be used in par ticular and for the decision on a problem (1.1)-(1.3) as a test example, when takes place:

$$\begin{cases} \upsilon_{i,0} \equiv \lambda_i V^0(\xi), i=\overline{1,3}, \\ \upsilon_i = \lambda_i V(\xi,t) + K_0(\xi,t) \equiv \Omega_i(\xi,t), \\ K_0(\xi,t) \equiv \sqrt{(\mu kt)^3}\exp(-\frac{\xi^2}{4k\mu t}), \\ \xi = \sum_{i=1}^{3}\gamma_i x_i, 0<\lambda_i, \sum_{i=1}^{3}\gamma_i = 0, \sum_{i=1}^{3}\lambda_i\gamma_i = 0, \\ \gamma_1 = \gamma_2 = 1, \gamma_3 = -2, \lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 2, \\ k = \sum_{i=1}^{3}\gamma_i^2, d_0 = \sum_{i=1}^{3}\lambda_i, (k=6, d_0 = \sum_{i=1}^{3}\lambda_i = 6), \\ \frac{\partial}{\partial x_i}\sum_{i=1}^{3}\upsilon_i = \sum_{i=1}^{3}[\lambda_i\gamma_i V_\xi + \gamma_i K_{0\xi}] = 0, \\ \sum_{i=1}^{3}\gamma_i\Omega_i = \sum_{i=1}^{3}[\lambda_i\gamma_i V(\xi,t) + \gamma_i K_0(\xi,t)] = 0, \end{cases}$$

that

$$\lambda_i V_t + 2\sqrt{(\mu k)^3 t} \times \exp(-\frac{\xi^2}{4k\mu t}) = f_i(\xi,t) - \\ -\frac{1}{\rho}\gamma_i P_\xi + \mu k \lambda_i V_{\xi^2}, i = \overline{1,3}. \quad (3.21)$$

or

$$\begin{cases} V_t = \frac{1}{d_0}\Phi_0(\xi,t) + \mu k V_{\xi^2}, \\ \frac{1}{\rho}P = -\int_\xi^\infty F_1(\eta,t)d\eta, \\ \frac{1}{\rho}P_\xi = F_1 \equiv \frac{1}{k}\sum_{i=1}^3 \gamma_i f_i, \\ \sum_{i=1}^3 \gamma_i F_1 = 0, \\ F_1 = \frac{1}{6}(f_1 + f_2 - 2f_3), \\ \Phi_0 \equiv -6\sqrt{(\mu k)^3 t} \times \exp(-\frac{\xi^2}{4k\mu t}) + \sum_{i=1}^3 f_i. \end{cases}$$

Then

$$\begin{cases} V = \frac{1}{\sqrt{\pi}}\int_R \exp(-\tau^2)V^0(\xi + 2\tau\sqrt{\mu k t})d\tau + \\ +\frac{1}{d_0\sqrt{\pi}}\int_0^t\int_R \exp(-\tau^2)\Phi_0(\xi + 2\tau\sqrt{\mu k(t-s)};s)\times \\ \times d\tau ds \equiv H(\xi,t), \\ \upsilon_i = \lambda_i H(\xi,t) + K_0(\xi,t) \equiv H_i(\xi,t), i = \overline{1,3}. \end{cases}$$

Remark 3. As $\tilde{V}^0(\xi) \in C^3(R)$, that limitedly the decision of a problem of Navier-Stokes (1.1) - (1.3) with a condition (B) it is possible to prove limitation and in $W_\lambda^2(D_0)$ - weight space of type of Sobolev:

$$\begin{cases} \|v\|_{W_\lambda^2} = \sum_{i=1}^3 \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}}, \\ \tilde{W}^2_{(\upsilon_i,\lambda)} = \{(\xi,t) \in D_0 : \upsilon_{i\xi^k}, \upsilon_{it} \in L_\lambda^2(D_0), k = \overline{0,3}\}, \\ \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} = \{\sum_{k=0}^3 \sup_R \int_0^{T_0} \lambda(t)[\upsilon_{i\xi^k}(\xi,t)]^2 dt + \\ +\sup_R \int_0^{T_0} \lambda(t)|\upsilon_{it}(\xi,t)|^2 d\xi dt\}^{\frac{1}{2}}, i = \overline{1,3}, \end{cases}$$

as $\tilde{V}(\xi,t) \in W_\lambda^2(D_0), \tilde{V} = (V_1, V_2, V_3)$.

4. Conclusions

I. From the received results follows that system Navier-Stokes (1.1) in the conditions of (1.2), (1.3), (A_1)-(A_3) can have the analytical unique is conditional-smooth decision. At least, such decision answers to a mathematical question, and possibility to construct the decision of a problem of Navier-Stokes (1.1)-(1.3) for an incompressible liquid with viscosity.

II. Results of the theorem 1 and 5 can be applied to a problem of Navier-Stokes of an incompressible fluid with viscosity, when $v \in R^n, x \in R^n, t \in R_+ = [0,\infty)$.

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